CH 5 : Op Amp

CH 6 : - general facts for C and L $i = C\frac{dv}{dt}$ $v = L\frac{di}{dt}$

- "dc conditions"

CH 7, 8, 9 : sinusoidal steady-state analysis (for (second-order) circuit driven by sinusoidal source) (phasor, ac)

CH 10 : first-order circuits (RC (and RL))

will see that the steady-state of this is the same as the "dc conditions"

CHAPTER 8

Sinusoidal Steady-State Analysis

8.1. General Approach

In the previous chapter, we have learned that the steady-state response of a circuit to sinusoidal inputs can be obtained by using phasors. In this chapter, we present many examples in which nodal analysis, mesh analysis, Thevenin's theorem, superposition, and source transformations are applied in analyzing ac circuits.

8.1.1. **Steps to analyze ac circuits, using phasor domain:**

Step 1. Transform the circuit to the phasor or frequency domain.

- Not necessary if the problem is specified in the frequency domain.

Step 2. Solve the problem using circuit techniques (e.g., nodal analysis, mesh analysis, Thevenin's theorem, superposition, or source transformations)

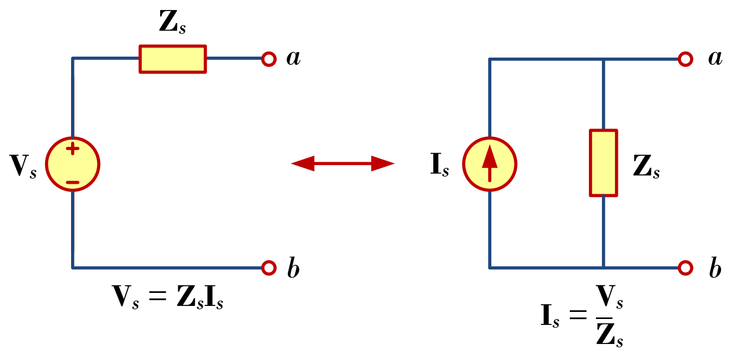
- The analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.

Step 3. Transform the resulting phasor back to the time domain.

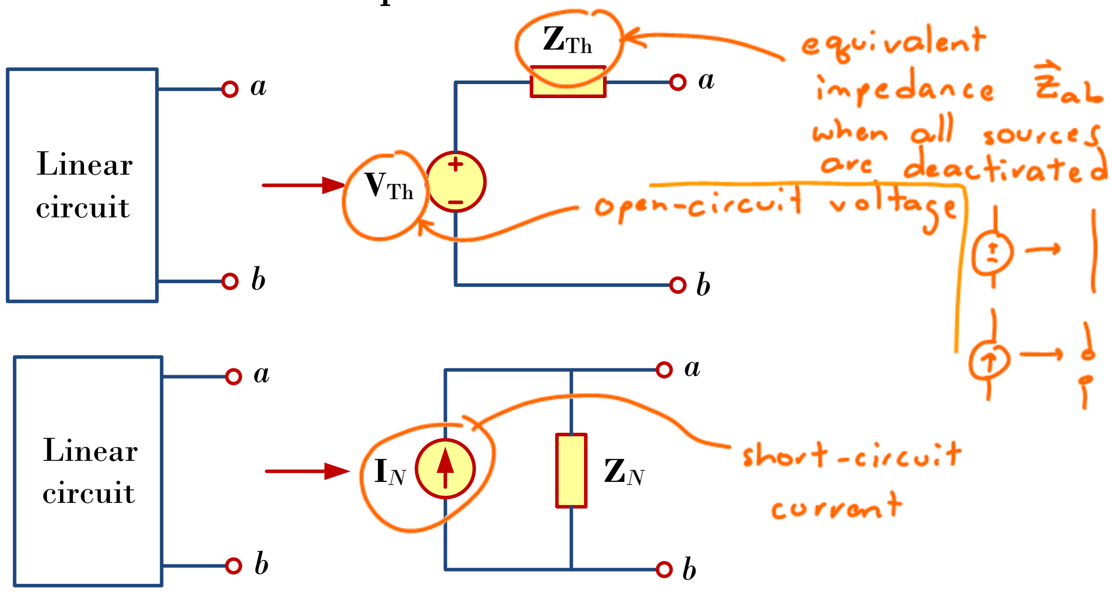
8.1.2. The **superposition theorem** applies to ac circuits the same way it applies to dc circuits. This is the case when all the sources in the circuit operate at the same frequency.

8.1.3. **Source transformation**:

$$\mathbf{V_s} = \mathbf{Z_s}\mathbf{I_s}, \quad \mathbf{I_s} = \frac{\mathbf{V_s}}{\mathbf{Z_s}}.$$



8.1.4. **Thevenin and Norton Equivalent circuits**:



$$\mathbf{V_{Th}} = \mathbf{Z_N}\mathbf{I_N}, \quad \mathbf{Z_{Th}} = \mathbf{Z_N}$$

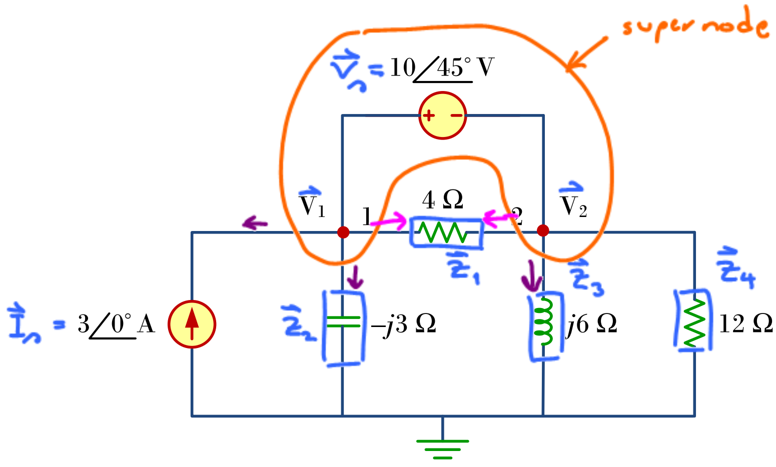
EXAMPLE 8.1.5. Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit below using nodal analysis.

Supernode:

$\vec V_s = \vec V_1 - \vec V_2$

$\Downarrow$

$\vec V_1 = \vec V_s + \vec V_2$



$\vec V_s = 10 \angle 45^\circ$ V

$\vec I_s = 3$ A

$\vec Z_1 = 4\,\Omega$

$\vec Z_2 = -3j\,\Omega$

$\vec Z_3 = 6j\,\Omega$

$\vec Z_4 = 12\,\Omega$

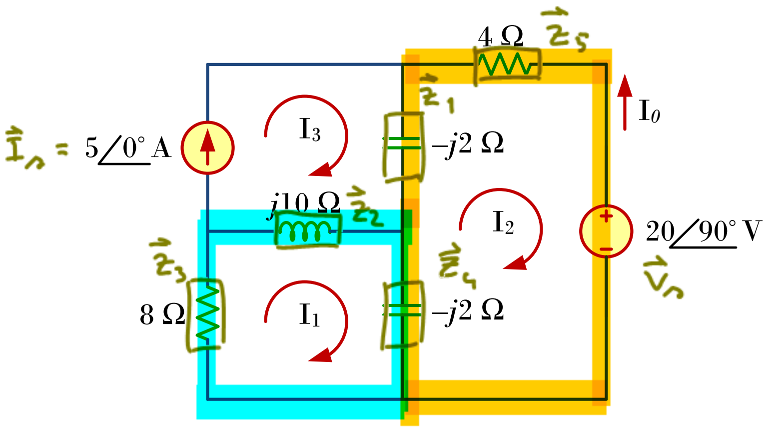
At supernode

KCL: $-\vec I_s + \frac{\vec V_1 - 0}{\vec Z_2} + \cancel{\frac{\vec V_1 - \vec V_2}{\vec Z_1}} + \cancel{\frac{\vec V_2 - \vec V_1}{\vec Z_1}} + \frac{\vec V_2 - 0}{\vec Z_3} + \frac{\vec V_2 - 0}{\vec Z_4} = 0$

$$\frac{\vec V_s + \vec V_2}{\vec Z_2} + \frac{\vec V_2}{\vec Z_3} + \frac{\vec V_2}{\vec Z_4} = \vec I_s \Rightarrow \vec V_2 = \frac{\vec I_s - \vec V_s / \vec Z_2}{\frac{1}{\vec Z_2} + \frac{1}{\vec Z_3} + \frac{1}{\vec Z_4}} = 31.4 \angle -87^\circ \text{ [V]}$$

$\vec V_1 = \vec V_s + \vec V_2 = 25.8 \angle -70.5^\circ$ [V]

EXAMPLE 8.1.6. Determine current $\mathbf{I}_o$ in the circuit below using mesh analysis.



$\vec V_s = 20 \angle 90^\circ = 20j$ [V]

$\vec I_s = 5 \angle 0^\circ = 5$ [A]

$\vec Z_1 = -2j\,\Omega$

$\vec Z_2 = 10j\,\Omega$

$\vec Z_3 = 8\,\Omega$

$\vec Z_4 = -2j\,\Omega$

$\vec Z_5 = 4\,\Omega$

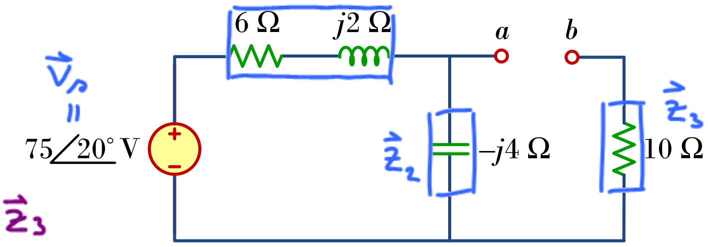
Mesh 3: $\vec I_3 = \vec I_s$

Mesh 1: $-\vec I_1 \vec Z_3 - (\vec I_1 - \vec I_3)\vec Z_2 - (\vec I_1 - \vec I_2)\vec Z_4 = 0$

Mesh 2: $-(\vec I_2 - \vec I_1)\vec Z_4 - (\vec I_2 - \vec I_3)\vec Z_1 - \vec I_2 \vec Z_5 - \vec V_s = 0$

$\Rightarrow \vec I_2 = 5 - 3.53j \quad \Rightarrow \vec I_o = -\vec I_2 = -5 + 3.53j = 6.12 \angle 144.8^\circ$

EXAMPLE 8.1.7. Find the Thevenin equivalent at terminals a-b of the circuit below.



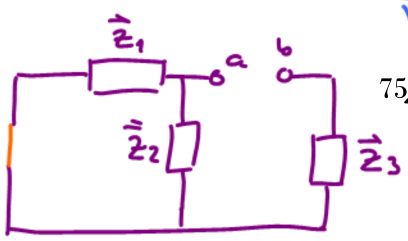
$\vec{V}_s = 75\angle 20^\circ$

$= 70.5 + 25.67j$

$\vec{Z}_1 = 6 + 2j\ \Omega$

$\vec{Z}_2 = -4j$

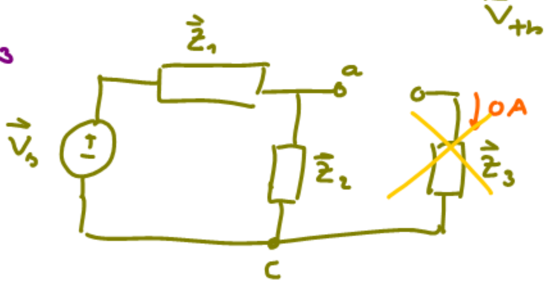
$\vec{Z}_3 = 10\ \Omega$

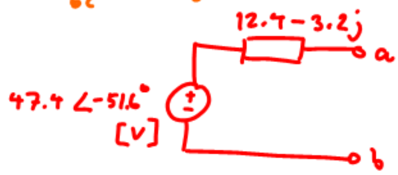


$$\vec{V}_{th} = \vec{V}_{ab} = \vec{V}_{ac} \overset{\text{VDF}}{=} \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}\vec{V}_s = 47.4\angle -51.6^\circ\ [\text{V}]$$

$$\vec{V}_{bc} = 0 \times \vec{Z}_3 = 0$$

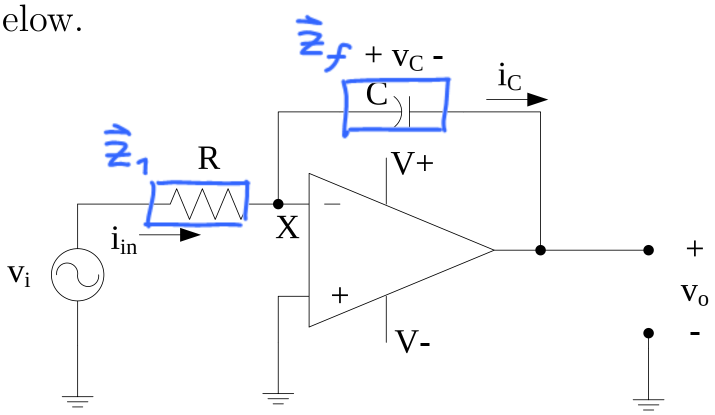
$$\vec{Z}_{th} = \vec{Z}_{ab} = (\vec{Z}_1 /\!/ \vec{Z}_2) + \vec{Z}_3 = \frac{\vec{Z}_1\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} + \vec{Z}_3 = 12.4 - 3.2j\ \Omega$$





EXAMPLE 8.1.8. **Op Amp AC Circuits**: Find the (closed-loop) gain of the circuit below.

Recall: If we write R_1 and R_f instead of $\vec{Z}_1$ and $\vec{Z}_f$, then this is an inverting amp.



We know that

$$V_o = -\frac{R_f}{R_1} V_i \iff \vec{V}_o = -\frac{\vec{Z}_f}{\vec{Z}_1}\vec{V}_i$$

$$\frac{\vec{V}_o}{\vec{V}_i} = -\frac{\vec{Z}_f}{\vec{Z}_1} = -\frac{\frac{1}{j\omega C}}{R} = \boxed{j\left(\frac{1}{\omega C R}\right)} = j\left(\frac{1}{(2\pi f) CR}\right) = (1\angle 90^\circ)\left(\frac{1}{2\pi f RC}\right)$$

In time domain,

$$\vec{V}_i = V_m \angle \phi \iff v_i(t) = V_m \cos(\omega t + \phi)$$

$$\vec{V}_o = (1\angle 90^\circ)\left(\frac{1}{2\pi f RC}\right)(V_m \angle \phi)$$

$$= \frac{V_m}{2\pi f RC}\angle(90^\circ + \phi) \iff v_o(t) = \frac{V_m}{2\pi f RC}\cos(\omega t + \phi + 90^\circ)$$